What is Informatics?
You have 100 people in your group.
Could there be 25 girls and 35 guys in this group?
It is right? Why?

It all depends on the number system in which we count.

What is "position"?
What associations do you have?

What associations do you have when you ask: What is "position"?

Examples of responses:

opinion;

judgments;

Dance position;

Point of view;

Equipped place;

Human rights;

Any pose;

numbers, etc.

Position (from Latin Positio - position)

Regulation - a regulatory or local legal act that defines the basic rules for the organization and activities of state bodies, structural divisions of the body, as well as institutions, organizations and enterprises (branches) that are subordinate to them, temporarily created commissions, groups, bureaus, and etc.

Geographic location - the geospatial relationship of a specific object to the external environment, the elements of which have or may have a significant impact on it

Number system - the number of characters, the sequence of which allows you to encode one or another piece of information

Numbers system

Positional number systems

decimal	binary	octal	hexadecimal	Binary decimal
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Non-positional number systems

Number System Classification

Positional/ Weighted Number System	Non-Positional/ Non-Weighted Number System

Positional/ Weighted Number System

- Decimal
- Octal
- Binary
- Hexadecimal
- BCD
- 8-4-2-1 Code

Non-Positional/ Non-Weighted Number System

- Excess-3 Code
- Cyclic Code
- Roma Code
- Gray Code

Non-positional Number Systems

- **Characteristics**
  - Use symbols such as I for 1, II for 2, III for 3, IIII for 4, IIIII for 5, etc
  - Each symbol represents the same value regardless of its position in the number
  - The symbols are simply added to find out the value of a particular number
- **Difficulty**
  - It is difficult to perform arithmetic with such a number system

Roman Numerals

I
II
III
IV
V

VI
VII
VIII
IX
X

1	5	10	50	100	500	1000
I	V	X	L	C	D	M

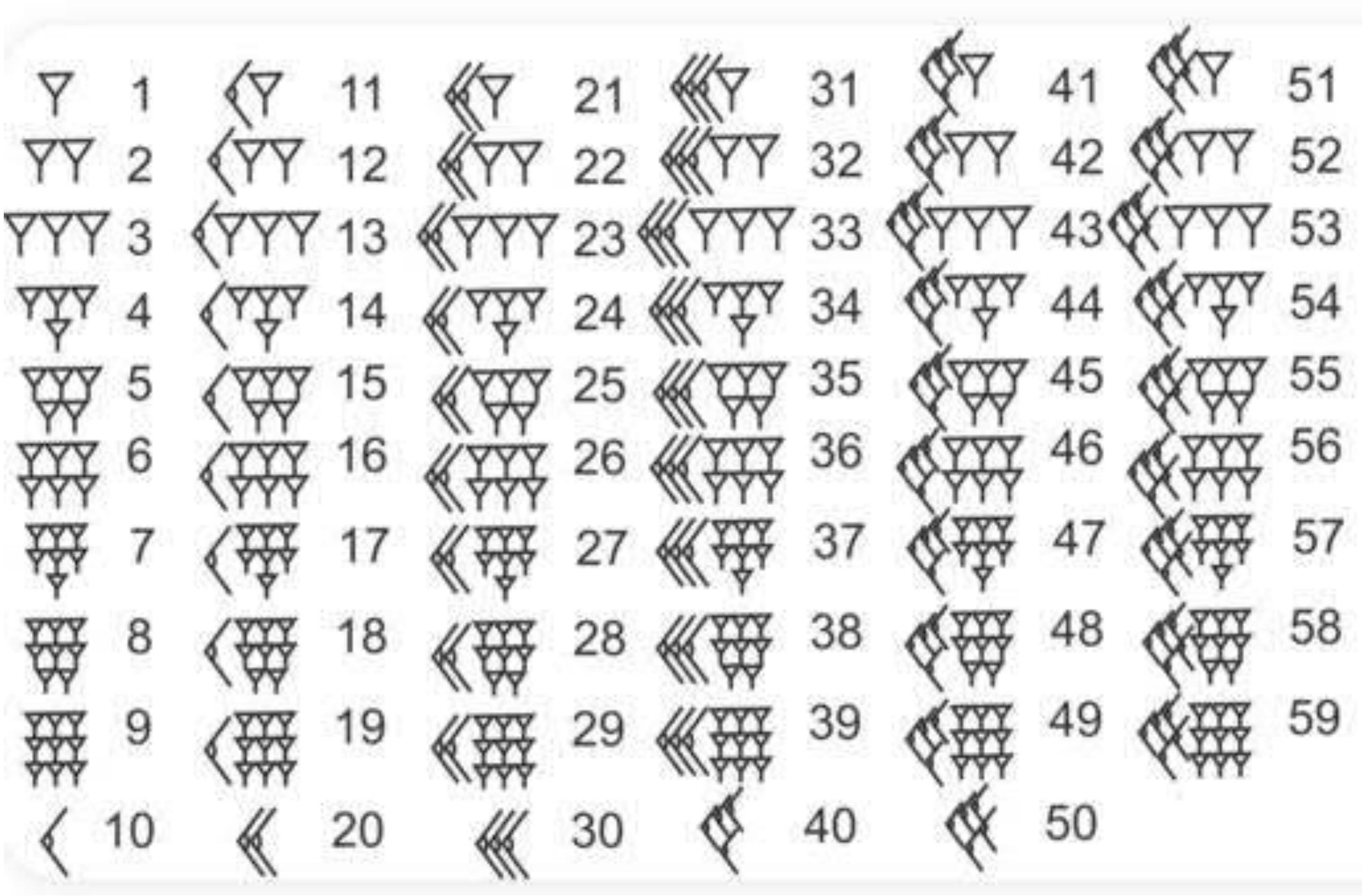
87 LXXXVII

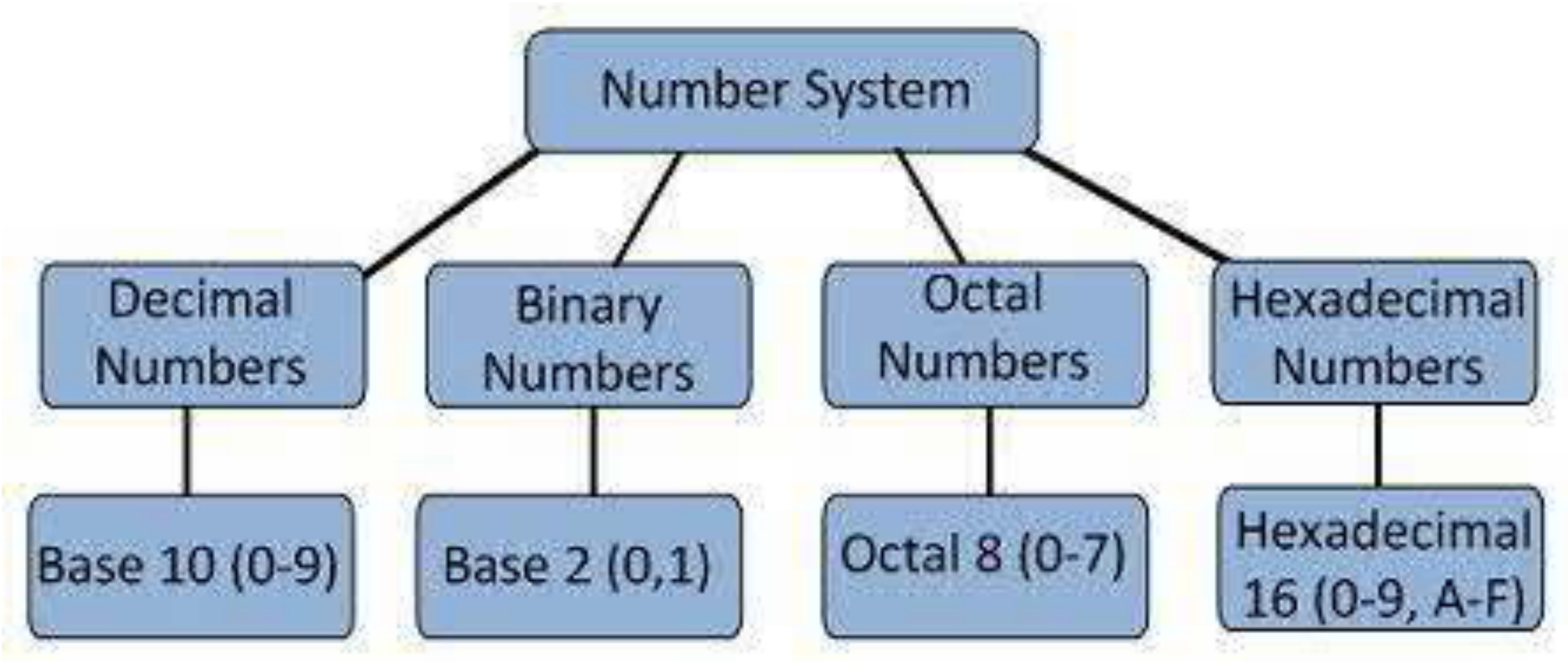
Non-Positional Number System

- ✓ Symbol represents the value regardless of its position.
- ✓ Difficult to perform arithmetic operation.
- ✓ For example:-

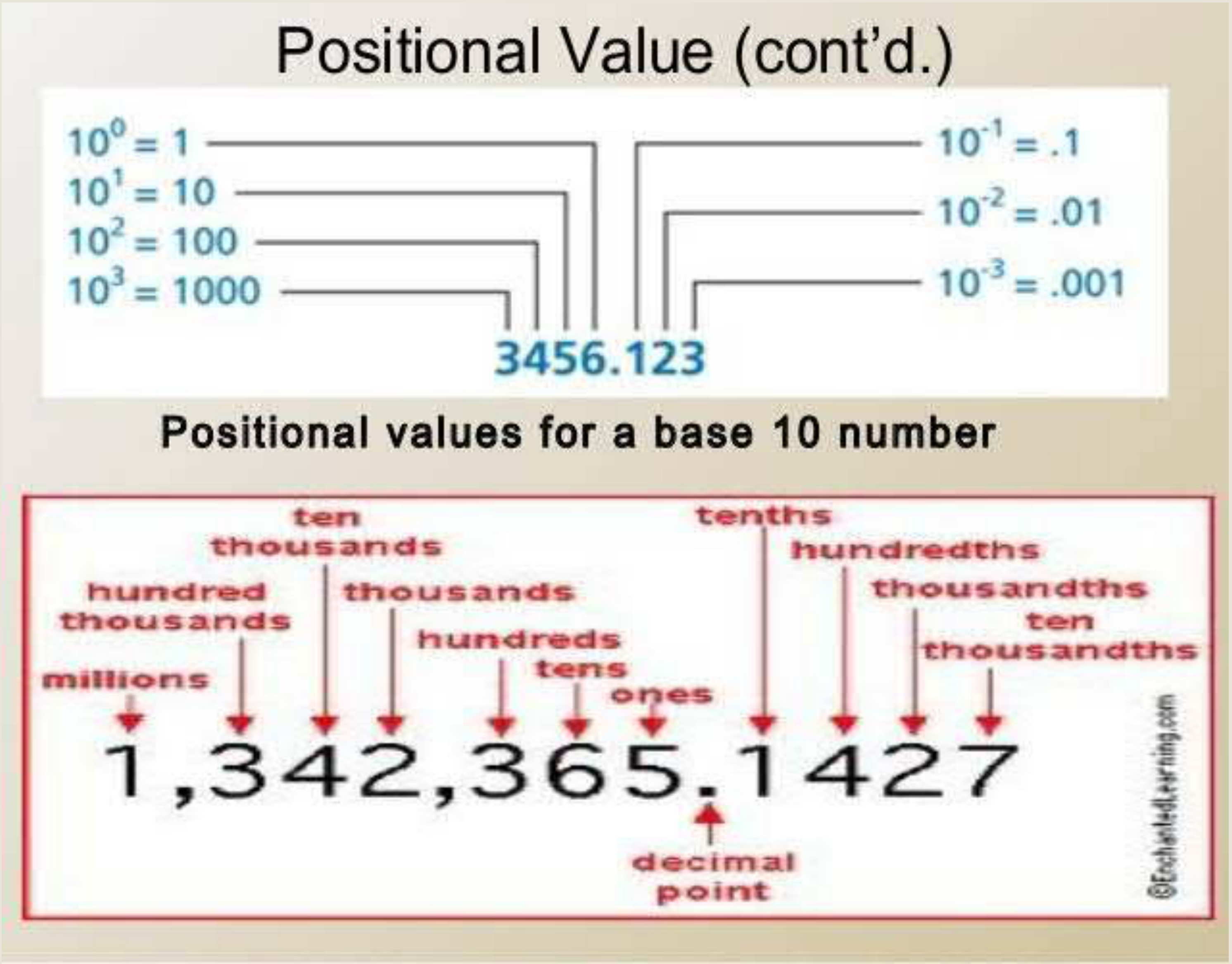
I, II, III, IV, V, VI, VII, VIII, IX, X
XI,XII, XIII, XIV, XV, XVI, XVII,
XVIII, XIX, XX

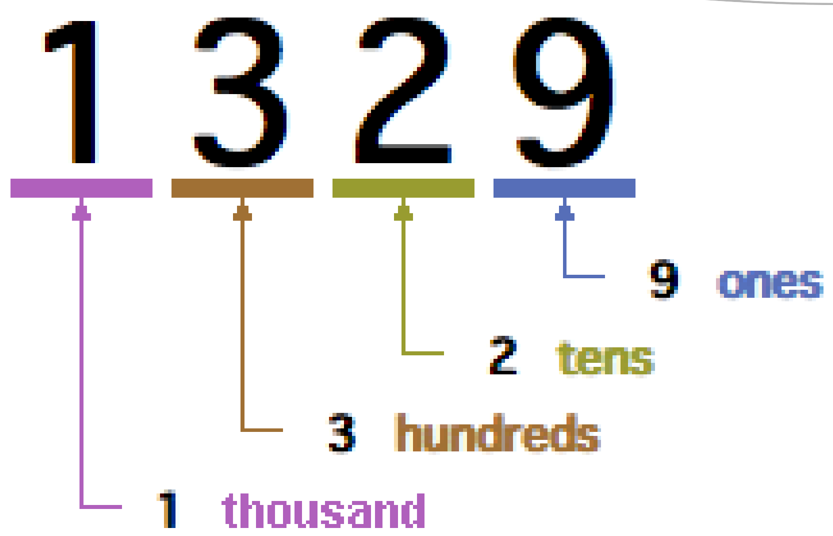
Babylonian numerals




Number System
Decimal Numbers
Binary Numbers
Octal Numbers
Hexadecimal Numbers
Base 10 (0-9)
Base 2 (0,1)
Octal 8 (0-7)
Hexadecimal 16 (0-9, A-F)

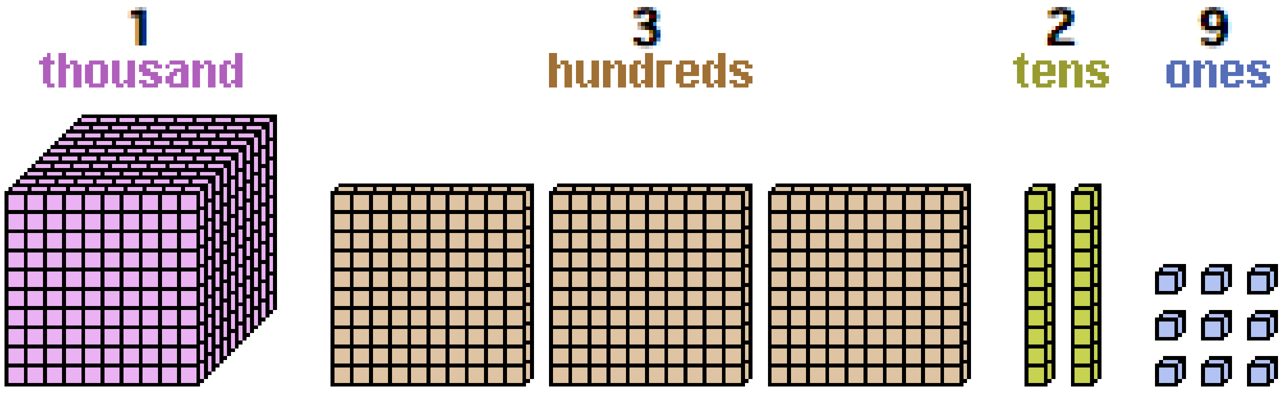
1 1 2 5


Positional Value (cont'd.)
10^0 = 1
10^1 = 10
10^2 = 100
10^3 = 1000
10^-1 = .1
10^-2 = .01
10^-3 = .001
3456.123
Positional values for a base 10 number
millions
hundred thousands
ten thousands
thousands
hundreds
tens
ones
tenths
hundredths
thousandths
ten thousandths
1,342,365.1427
decimal point
©EnchantedLearning.com



1329 : one thousand, three hundred and twenty-nine

$1329 = (1 \times 1000) + (3 \times 100) + (2 \times 10) + (9 \times 1)$



In modern mathematics, the positional decimal number system is used. This is because people learned to count using their fingers, and a person has five fingers on each hand. There would be six of them, we would consider not tens, but dozens.

The number system alphabet is a set of numbers used to write numbers in a given number system.

Base of the number system = Code base - the size of the alphabet, the number of characters used to display a number in this number system - the smallest number of characters that is used to encode information

Code length - the number of characters, the sequence of which allows you to encode this or that form of information

In non-positional number systems, the value of a number is represented by this or that symbol, regardless of the place that this symbol occupies in the number record

In the positional number system, the value of the number represented by this or that symbol depends not only on the value of the symbol itself, but also on the position occupied by this symbol of the number

Number systems, Operations, and Codes

1- Decimal Numbers

The decimal number system has ten digits .

These are : 0 , 1 , 2 , 3, 4 , 5 , 6 , 7 , 8 , 9 .

The decimal number system has the base = 10 →

$$10^2 \; 10^1 \; 10^0 . 10^{-1} \; 10^{-2} \; 10^{-3} \ldots$$

↑ Decimal point

Example -1-

$$47 = (4 \times 10^1) + (7 \times 10^0)$$
$$= (4 \times 10) + (7 \times 1) = 40 + 7$$

Example -2-

$$568.23 = (5 \times 10^2) + (6 \times 10^1) + (8 \times 10^0) + (2 \times 10^{-1}) + (3 \times 10^{-2})$$
$$= (5 \times 100) + (6 \times 10) + (8 \times 1) + (2 \times 0.1) + (3 \times 0.01)$$
$$= 500 + 60 + 8 + 0.2 + 0.03$$

Example -3- : Express the decimal number 897.9 as a sum of the value of each digit.

2- Binary Numbers

The binary number system has two digits (bits) .

These are : 0 , 1.

The binary number system has the base = 2

$$2^{n-1} \ldots 2^3\, 2^2\, 2^1\, 2^0 . 2^{-1}\, 2^{-2} \ldots 2^{-n}$$

Binary point

The weight of a bit increases from right to left in a binary whole number

DECIMAL NUMBER	BINARY NUMBER			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

Octal Number System

Base (Radix)	8
Digits	0, 1, 2, 3, 4, 5, 6, 7
e.g.	1623_8

1	6	2	3
$8^3=512$	$8^2=64$	$8^1=8$	$8^0=1$

The digit 2 in the second position from the right represents the value 16 and the digit 1 in the fourth position from the right represents the value 512.

4- Hexadecimal Numbers

The hexadecimal number system has 16 digits.

These are : 0 , 1, 2 , 3, 4 , 5 , 6, 7 , 8 , 9, A , B , C , D , E , F

The hexadecimal system has the base = 16

DECIMAL	BINARY	HEXADECIMAL
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

The main property of positional number systems is that any number can be represented as the sum of a series, decomposed by the degree of the base

Binary number system

The computer processes numerical, textual, graphic, audio and video information.

The question arises: "How, in what way, the computer processes information so different in human perception?"

All of these types of information are encoded in a sequence of electrical impulses:

there is an impulse (1),

no impulse (0),

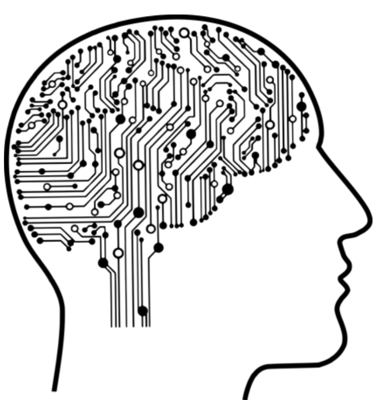
that is, sequences of zeros and ones. Such encoding of information in a computer is called binary coding; a computer uses a binary number system.

Number series (Alphabet) - 0, 1,

Series (alphabet) size = base: 2

decimal	octal	hexadecimal	binary
0	0	0	0000
1	1	1	0001
2	2	2	0010
3	3	3	0011
4	4	4	0100
5	5	5	0101
6	6	6	0110
7	7	7	0111
8	10	8	1000
9	11	9	1001
10	12	A	1010
11	13	B	1011
12	14	C	1100
13	15	D	1101
14	16	E	1110
15	17	F	1111

Please note that in every number system there is an entry 10, but these numbers are not equal to each other. Therefore, it is imperative to indicate in which number system we use the number. To do this, write in the subscript the base of the number system



In digital technology, all information, regardless of its nature, is presented in numerical form, and only positional number systems are used. In these systems, any integer positive n-bit number is written as a sequence of n digits

$$X_{n-1}X_{n-2}..X_1X_0.$$

number a (0, 1, 2, ..., a - 1),

adopted to represent numbers that determine the base of the number system. The contribution of a digit to a number depends both on this base and on the position (rank) it occupies in the sequence of digits. The digit xk comes with weight ak and means $x_k a_k$, and the whole sequence of numbers $X_{n-1}X_{n-2} \ldots X_1X_0$ expresses in the base a number system $x_{n-1}\, a^{n-1} + x_{n-2}\, a^{n-2} + \ldots + x_1\, a^1 + x_0\, a^0$.

The usual decimal system ($a = 10$) uses numbers 0, 1, 2, ... 9. Example,

$$3175 = 3 * 10^3 + 1 * 10^2 + 7 * l0^1 + 5 * 10^0.$$

In computing, the binary number system has become predominant, for which are used the numbers 0 and 1

The Binary digit - the smallest amount of information

The Binary digit is called a bit. A sequence of binary digits $X_{n-1}X_{n-2} \dots X_1X_0$ serves as a binary number record

$$x_{n-1} 2^{n-1} + x_{n-2} 2^{n-2} + \dots + x_1 2^1 + x_0 2^0.$$

Octal and hexadecimal are the most commonly used number systems. In the octal system, numbers are represented by the same characters as in the decimal system, and in the hexadecimal system, six more characters are added to them A, B, C, D, E, F, which correspond to decimal numbers 10, 11, 12, 13, 14, 15.

If you need to specify the base of the number system, the number is recorded with an index. For example:

$10110_2=(1*2^4+1*2^3+0*2^2+1*2^1+0*2^0) =26_{10};$

$5327_8=(5*8^3+3*8^2+2*8^1+7*8^0) =2775_{10};$

$2DF9_{16}=(2*16^3+13*16^2+15*16^1+9*16^0) =11769_{10}.$

To convert a number from any number system to decimal, it is enough to calculate the value of the corresponding polynomial, substituting the decimal values of the digits and the base of the number system into it. It is convenient to perform calculations according to Horner's scheme, based on the representation of the polynomial in the form

$$(\ldots((x_{n-1}\,a + x_{n-2})a + x_{n-3})a + \ldots + x_1)a + x_0\,a,$$

with the help of which various operations are carried out in decimal and other number systems.

In the general case, when a number has a fractional part, the latter is separated from the integer frequent by a separating character - a dot or comma:

where n - digits of the integer part

and m - digits of the small part

$$x_{n-1}\, x_{n-2} \ldots x_1\, x_0,\, x_{-1}\, x_{-2} \ldots x_{-m}\ ,$$

which corresponds to the number :

$$x_{n-1}\, a^{n-1} + x_{n-2}\, a^{n-2} + x_1\, a^{1} + x_0\, a^0 + x_{-1}\, a^{-1} + x_{-2}\, a^{-2} + x_{-m}\, a^{-m}$$

The expression of any number in the decimal system is reduced to the calculation of its polymember representation, for example;

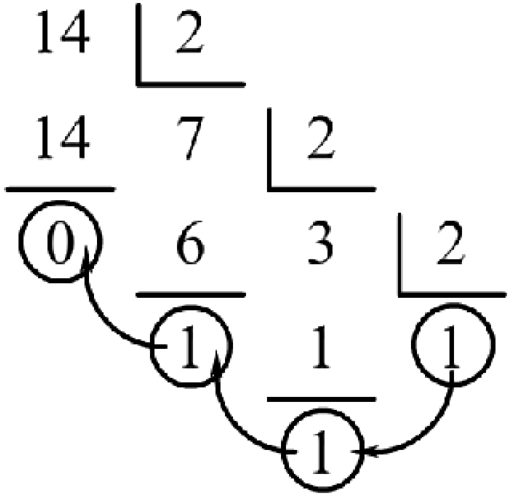
$$405{,}37_8 = (4*8^2+0*8^1+ 5 * 8^\circ + 3*8^{-1} + 7 *8^{-2})_{10} = 261{,}140625_{10}.$$

Arithmetic operations on numbers in any number system are performed according to the rules that are used in the decimal system.

Rules for converting from decimal to binary:

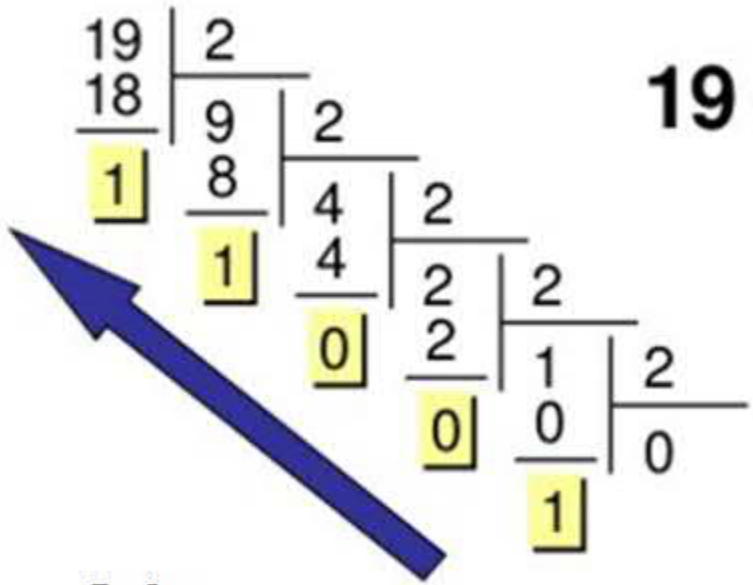
1. Divide the decimal number by 2. Get the quotient and remainder.
2. Divide the quotient by 2. Get the quotient and the remainder.
3. Perform division until the last quotient is less than 2.
4. Write the last quotient and all residuals in reverse order. The resulting number will be the binary code of this decimal number.

Example,
N_{10}=14



Answer : 1110_2

10 → 2



$$19 = 10011_2$$

Numbers system

2 → 10

4 3 2 1 0 Index

$$10011_2 = 1\cdot2^4 + 0\cdot2^3 + 0\cdot2^2 + 1\cdot2^1 + 1\cdot2^0$$

$$= 16 + 2 + 1 = 19$$

Binary Numbers to Decimal

0 -1 -2 -3 -4 ← power of 2 ↓

$$0.1011_2 = 1 \times 2^{-1} \longrightarrow 0.5$$

$$0 \times 2^{-2} \longrightarrow 0$$

$$1 \times 2^{-3} \longrightarrow 0.125$$

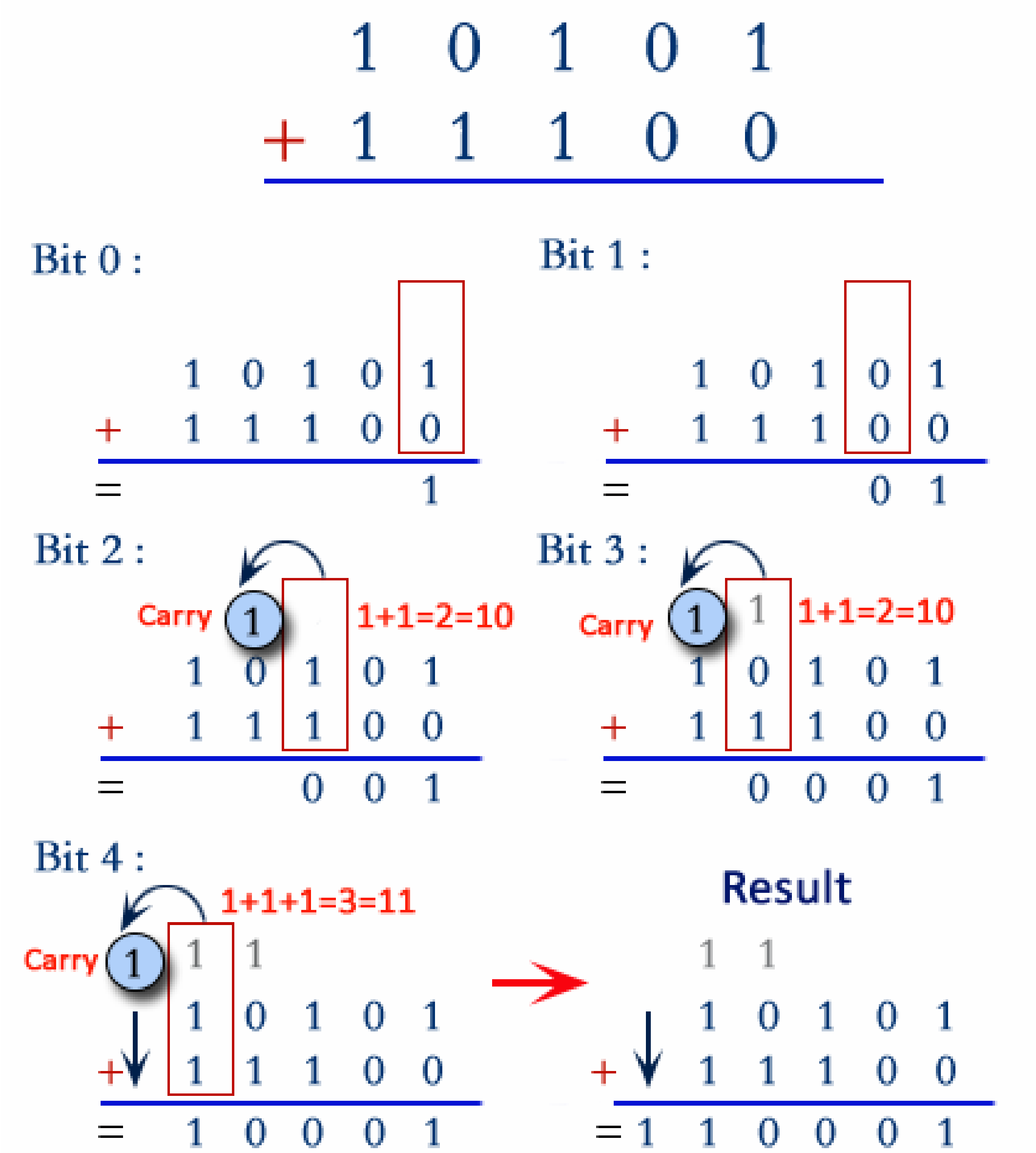
$$1 \times 2^{-4} \longrightarrow 0.0625$$

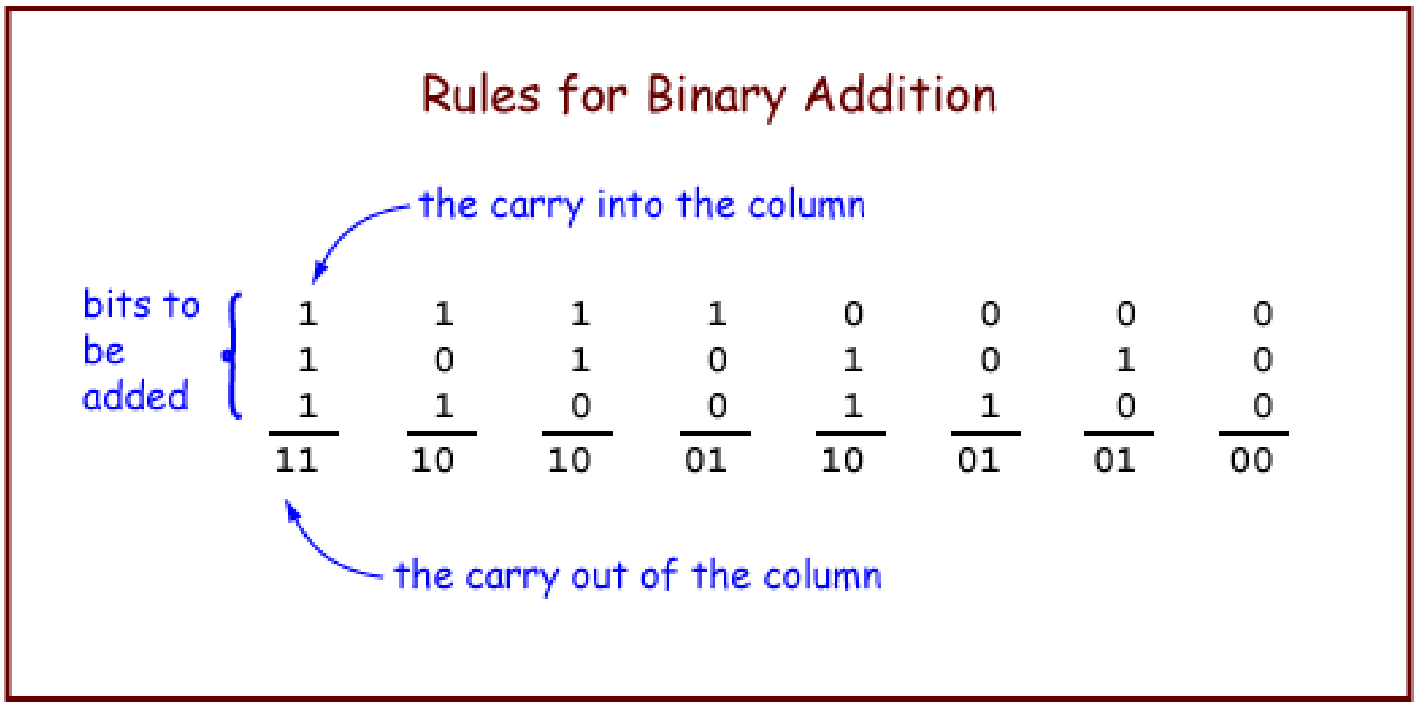
$$\longrightarrow 0.6875_{10}$$

Basic Arithmetic Operations with Binary Numbers

- Rules for Binary Addition
  - 1+1=0, with one to carry to the next place

				1
0	0	1	1	+1
+0	+1	+0	+1	+1
0	1	1	10	11


1 0 1 0 1
+ 1 1 1 0 0
Bit 0 :
1 0 1 0 1
+ 1 1 1 0 0
= 1
Bit 1 :
1 0 1 0 1
+ 1 1 1 0 0
= 0 1
Bit 2 :
Carry 1
1+1=2=10
1 0 1 0 1
+ 1 1 1 0 0
= 0 0 1
Bit 3 :
Carry 1
1
1+1=2=10
1 0 1 0 1
+ 1 1 1 0 0
= 0 0 0 1
Bit 4 :
1+1+1=3=11
Carry 1
1 1
1 0 1 0 1
+ 1 1 1 0 0
= 1 0 0 0 1
Result
1 1
1 0 1 0 1
+ 1 1 1 0 0
= 1 1 0 0 0 1


Rules for Binary Addition
the carry into the column
bits to be added
1 1 1 1 0 0 0 0
1 0 1 0 1 0 1 0
1 1 0 0 1 1 0 0
11 10 10 01 10 01 01 00
the carry out of the column

$$\begin{array}{r} 101101_2 \\ +\ \ 11111_2 \\ \hline \end{array}$$

$$\begin{array}{r} 10111_2 \\ +\ 101110_2 \\ \hline \end{array}$$

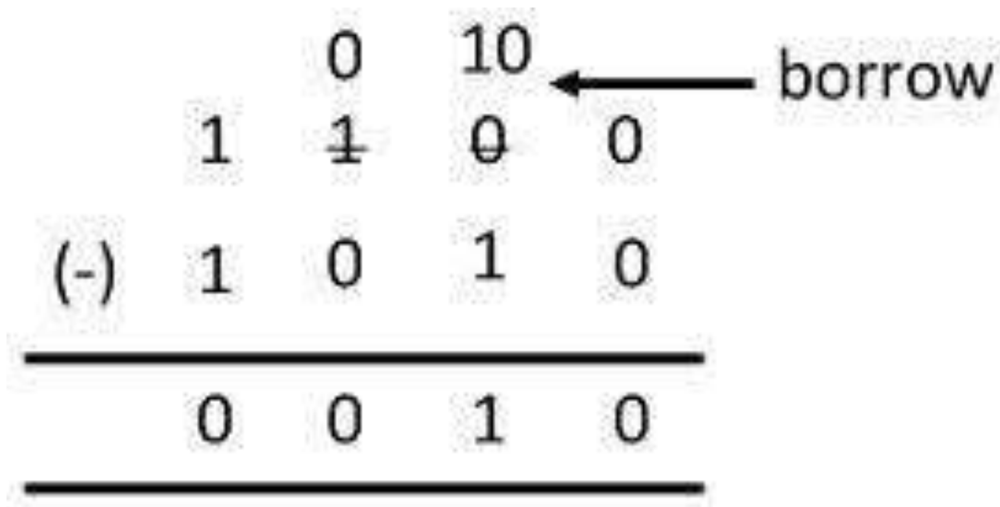
$$\begin{array}{r} 111011_2 \\ +\ \ 11011_2 \\ \hline \end{array}$$

$$\begin{array}{r} 111011_2 \\ +\ \ 10011_2 \\ \hline \end{array}$$

Basic Arithmetic Operations with Binary Numbers

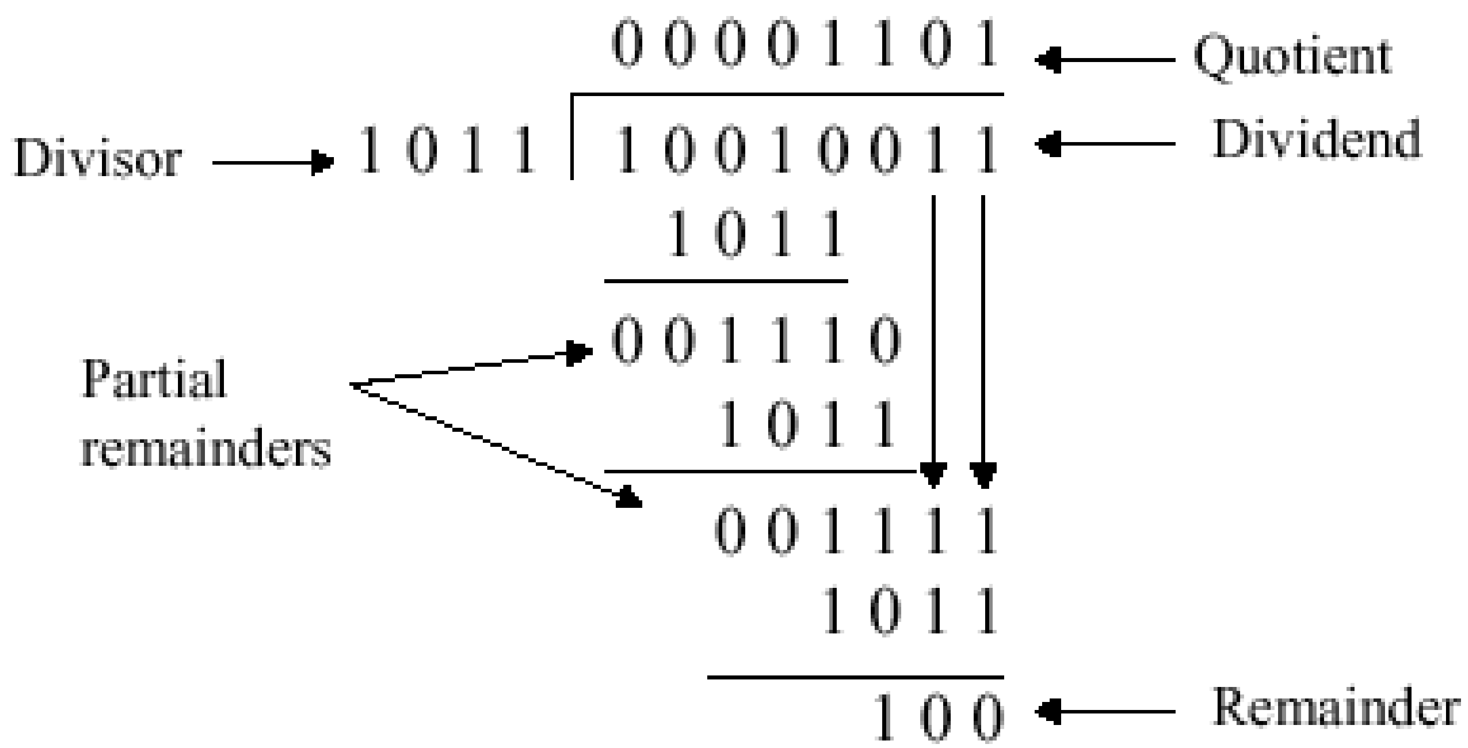
- Rules for Binary Subtraction
  - 1 - 0 = 1
  - 1 - 1 = 0
  - 0 - 0 = 0
  - 0 - 1 = 1, borrow 1 from the next most significant bit

M minuend
S subtrahend
D difference


0 10
borrow
1 1 0 0
(-) 1 0 1 0
0 0 1 0

$$
\begin{array}{r}
101101_2 \\
-\quad 11111_2 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
11011_2 \\
-\ 110101_2 \\
\hline
\end{array}
$$

				1	1	0	1	$(13)_{10}$	Multiplicand M
			×	1	0	1	1	$(11)_{10}$	Multiplier Q
				1	1	0	1		Partial products
			1	1	0	1			
		0	0	0	0				
	1	1	0	1					
1	0	0	0	1	1	1	1	$(143)_{10}$	Product P


0 0 0 0 1 1 0 1
Quotient
Divisor
1 0 1 1
1 0 0 1 0 0 1 1
Dividend
1 0 1 1
0 0 1 1 1 0
Partial
remainders
1 0 1 1
0 0 1 1 1 1
1 0 1 1
1 0 0
Remainder

The rule for converting from binary to decimal.

To converting from binary to decimal, you need to represent the binary number as a sum of powers of two and find its decimal value.

Example:

$$\left(N_2 \to N_{10}\right)$$

$$1001011_2 = ?$$

$$1001011_2 = 1\cdot 2^6 + 1\cdot 2^3 + 1\cdot 2^1 + 1\cdot 2^0 = 64 + 8 + 2 + 1 = 75$$

Answer: $N10 = 75_{10}$

Advantages of BST

- ✓ **Simple**
- ✓ **Efficient**
- ✓ **Dynamic**

- ✓ One of the most fundamental algorithms in CS
- ✓ The method of choice in many applications

Disadvantages of BST

- ✓ The shape of the tree depends on the order of insertions, and it can be degenerated.
- ✓ When inserting or searching for an element, the key of each visited node
  has to be compared with the key of the element to be inserted/found.
  Keys may be long and the run time may increase much.

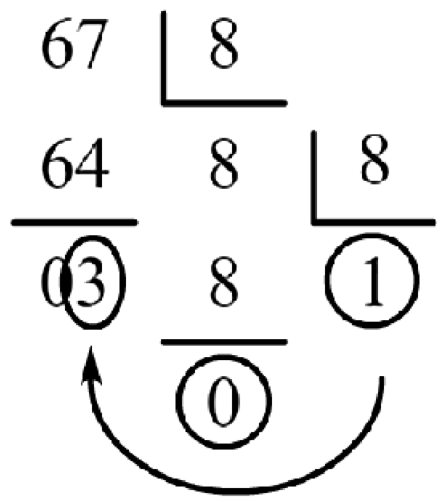
Rules conversion decimal to octal:

1. Divide the decimal number by 8. Get the quotient and remainder.
2. Divide the quotient by 8. Get the quotient and the remainder.
3. Perform division until the last quotient is less than 8.
4. Write the last quotient and all residuals in reverse order. The resulting number will be the binary code of this decimal number.

Example,

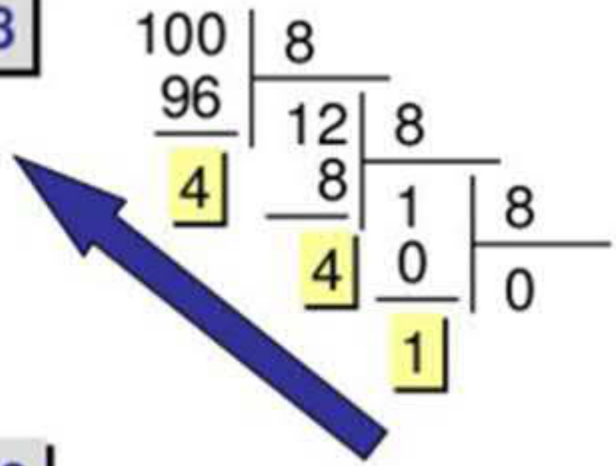
$$\left(N_{10} \to N_{8}\right)$$

$$67_{8} = ?$$



Answer :
$N_8 = 103_8$

10 → 8



$$100 = 144_8$$

Numbers system

8 → 10

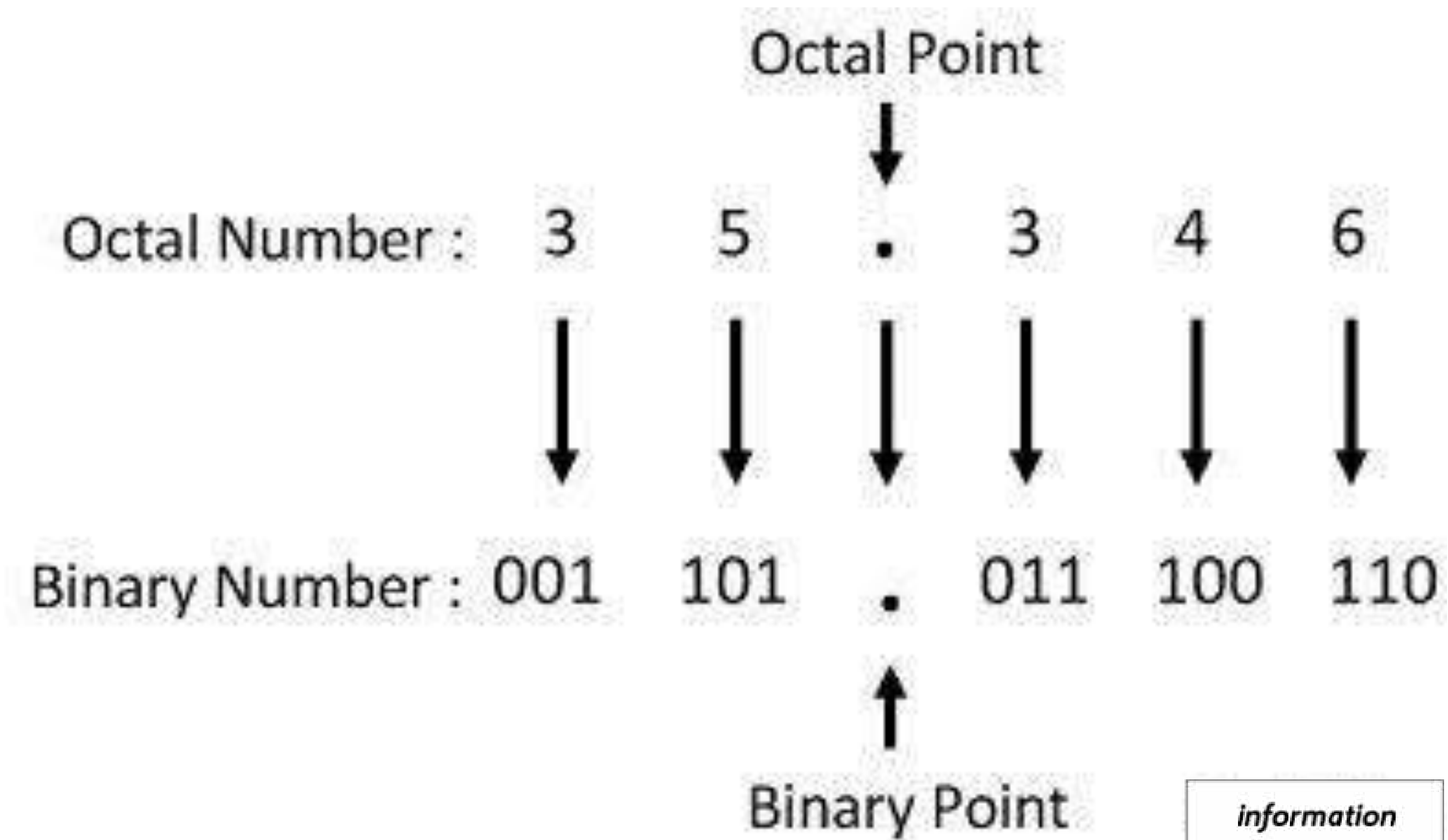
2 1 0 **Index**

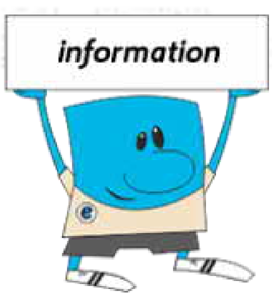
$$144_8 = 1 \cdot 8^2 + 4 \cdot 8^1 + 4 \cdot 8^0$$

$$= 64 + 32 + 4 = 100$$

X_{10}	X_8	X_2
0	0	000
1	1	001
2	2	010
3	3	011

X_{10}	X_8	X_2
4	4	100
5	5	101
6	6	110
7	7	111


Octal Point
Octal Number : 3 5 . 3 4 6
Binary Number : 001 101 . 011 100 110
Binary Point


information

$3467_8 =$

$2148_8 =$

$7352_8 =$

$1231_8 =$

Converting Binary to Octal

10001100101001_2

STEP ONE: Take the binary number and from right to left, group all placeholders in triplets. Add leading zeros, if necessary.

010 001 100 101 001

Rule of transition from decimal to hexadecimal system:

1. Divide the decimal number by 16. Get the quotient and remainder.
2. The quotient is again divided by 16. Get the quotient and the remainder.
3. Continue division until the last part is less than 16.
4. Write down the last quotient and all the rest in reverse order. The resulting number will be the hexadecimal code of this decimal number.

Example:

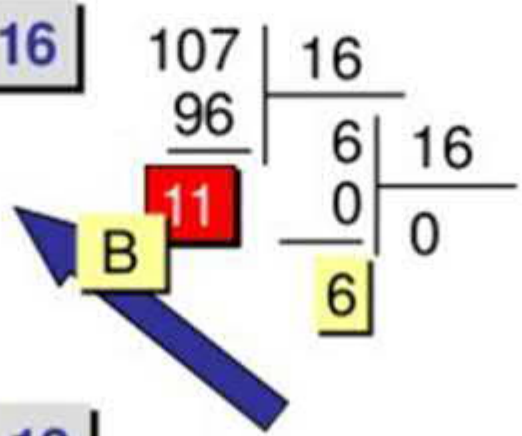
$$\left(N_{10} \rightarrow N_{16}\right)$$

$$42_{10} = ?$$

$$\begin{array}{r|l} 42 & 16 \\ \hline \underline{32} & \textcircled{2} \\ 10 & \end{array}$$

Answer :
N16=$2A_{16}$

10 → 16



$$107 = 6B_{16}$$

numbers system

16 → 10

2 1 0 **Index**

C

$$1C5_{16} = 1\cdot 16^2 + 12\cdot 16^1 + 5\cdot 16^0$$

$$= 256 + 192 + 5 = 453$$

$171 =$

$1BC_{16} =$

$206 =$

$22B_{16} =$

X_{10}	X_{16}	X_2
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111

X_{10}	X_{16}	X_2
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Hex to Binary Number Conversion

Convert the hexadecimal $9DB5_{16}$ to its binary equivalent.

1. Separate the digits of the given hexadecimal, if more than 1 digit.

9	D	B	5

2. Find the equivalent binary number for each digit of hex number, add 0's to the left if any of the binary number is shorter than 4 bits.

9	D	B	5
1001	1110	1011	0101

3. Write the all groups binary numbers together, maintaining the same group order provides the equivalent binary for the given hexadecimal.

1001111010110101

Result

$$9DB5_{16} = 1001111010110101_2$$

$C73B_{16} =$

$2FE1_{16} =$

To convert binary numbers into hexadecimals, you only have to make 4-bit groups and convert directly each group:

1 0 1 1 0 0 1 1 0 1 0 1 (binary)

B 3 5 (hex)

$1010101101010110_2 =$

$111100110111110101_2 =$

$110110110101111110_2 =$

$A35_{16} =$

$765_8 =$

CONCLUSION

Addition	Subtraction	Multiplication	Division
0 + 0 = 0 1 + 0 = 1 0 + 1 = 1 1 + 1 = 0 and carry 1	0-0=0 1-0=1 0-1=1 and carry 1 1-1=0	0 × 0 = 0 1 × 0 = 0 0 × 1 = 0 1 × 1 = 1	0 ÷ 1 =0 1 ÷ 0 = not defined 0 ÷ 0 =0 1÷ 1 =1
Example: 11 11 ——— 110 Here, 1+1 (right most) = 0 and its carry 1 is added to left columns as 1+1+1=11 Hence, 11+11=110	*Example:* 10 01 ——— 01 Here, 0-1 (right most) =1 because we take carry 2 from left column, and left remains 0. Hence, 10-01=01	*Example:* 1 1 1 0 ——— 0 0 1 1 × ——— 1 1 0 Tryout: (a) 1001 × 11 (b) 1100 × 101 (c) 1111 × 110 (d)1010 × 1001	*Example:* 11) 1 1 0(10 1 1 ——— × 0 0 Try out: (a) 111 ÷ 11 (b) 1100 ÷ 11 (c) 1001÷ 11 (d) 1011÷ 100 (e) 1111÷ 1011

Good to see you next time!